TITLE OF LESSON

Geometry Unit 1 Lesson 9 – Geometric Concepts: Theorems
Prove it! What's on the outside? What's on the inside? Of Geometry

TIME ESTIMATE FOR THIS LESSON

One class period

ALIGNMENT WITH STANDARDS

California – Geometry

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
2.0 Students write geometric proofs, including proofs by contradiction.
3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

MATERIALS

None

LESSON OBJECTIVES

- To discuss and suggest examples representing the idea of the postulate
- To discuss and suggest examples representing the idea of the theorem
- To distinguish among postulates, hypotheses, and theorems
- To begin to prove theorems in Geometry

FOCUS AND MOTIVATE STUDENTS

1) Homework Check – Stamp/initial complete homework assignment. Pass back graded work and have students place in the appropriate sections of their binders.
2) Agenda – Have students copy the agenda.
3) Present Homework – (5 minutes) Have each student read one of his hypotheses from the homework from Lesson 4. Have each explain how he might prove his hypothesis. There have now been three classes on proofs since this assignment was given. In this discussion, students should show increased understanding of the nature of a proof.

ACTIVITIES – INDIVIDUAL AND GROUP

1. Review: Postulates – (10 minutes) Review the 6 postulates. Ask students to state the postulates. It is acceptable for them to simply read the postulate out of their notebook. After each postulate has been read, have another student explain the postulates.

2. Discussion: Intersections – (5 Minutes) Write the following statement on the board: "If two different lines intersect, they intersect in one and only one point." Lead a discussion on what this means. Can you think of any examples from real life? What about two (straight) streets. How many ways do they intersect? When you say that I will meet you at 12^{th} and Broadway, do you know which intersection this is? What if there were two intersections that were known as 12^{th} and Broadway?

3. Lecture: Postulate or Theorem – (10 minutes) Is the statement above a postulate or a theorem? (Answer: I have seen it written both ways but I claim that it is a theorem since it can be proven from postulates using proof by contradiction. You haven't covered proof by contradiction yet. So you can introduce the idea at this point.) Proof by contradiction works as follows:

Let us say that we want to prove that *a* implies *b*. Instead of assuming *a* and proving that *b* must follow as we would in a straightforward proof we would instead assume a and also assume *not b*. By assuming *not b*, if we can show that *a* is not possible then we have shown that *not b* contradicts *a*. This is to say that we cannot both have *a* being true and *b* being false. This is the nature of the contradiction. The conclusion is that if *a* is true that *b* cannot be false and therefore must be true) If someone says that the statement in section activity 2 is a postulate have him or her argue that it cannot be proven from other postulates. If someone says that it is a theorem then ask him or her to prove it using the previous postulates. We may get a proof that is acceptable at this point. If not, suggest that if the lines intersect in more than one point—for instance, 2 points—then both line 1 and line 2 have at least points A and B in common. Now ask the question again. Postulate 3 will argue that these lines must be the same line but did we not assume that they were different lines so they can't be the same line and they can't be different lines at the same time. Point this out to the class and ask them what they think about that. Let them know that we have used a proof by contradiction to show that the intersection to two lines must one and only one point. Have one of the students draw the two intersecting lines on the board. Have them add the theorem and the proof to their binders.

4. Lecture: Proof – (15 minutes) Repeat the previous exercise with the following theorem: If a point lies outside a line, then exactly one plane contains both the line and the point. This will be a bit trickier than the previous theorem. If we choose any two points on the line and the third point we know by postulate 4 that there is exactly one plane and by postulate 5 we know that if two points lie in a plane then the line joining them lies in the plane so that the entire line lies in the plane so that no matter what two points we had chosen from the line we would have defined the same plane. Before explaining the proof, ask whether the statement is a theorem or a postulate. Have the students argue their assertions. See if someone can come up with a proof. Write any acceptable proof on the board stating the postulate and the reasons why this supports the argument. Discuss the meaning of this proof. Can the students think of any application to everyday life?

5. Discussion: Theorem – (5 minutes) Write the following statement on the board: "If two lines intersect, then exactly one plane contains both lines." Begin the discussion on this theorem. We will not get to a proof today, but the students should think about a proof for the next class.

6. Vocabulary – As you give the homework assignment, explain that the properties they will be coming up with are hypotheses. This is one of those rare words that forms its plural a bit differently from most words in English. Write this on the board for them to see the difference:

Singular	**Plural**
Hypothesis (high-poth' i-**sis**)	Hypotheses (high-poth' i-**seez**')
Basis (bay' **sis**)	Bases (bay' **seez**)
Base	Bases (bay' **siz**)
House	Houses (hou' ziz, **-siz**)

It may make it easier to write out the sounds as *siz* versus *seez*. The idea is that, to make a word that ends in *-sis* plural, we just change the *i* in between the two s's into an *e*. When we do this, the pronunciation of the end of the word changes from *-sis* to *-seez*. This is an important distinction because the *-ses* in most words that end in *-ses*, (e.g., bases, houses, cases), is pronounced *-siz*.

7. Homework Review – Explain the homework assignment. Field questions.

HOMEWORK

Think of two properties of lines that you think that we should be able to prove. Write them down.

Note: Many, if not most, will be wrong, but the point is to get them to start thinking about properties and about proving hypotheses. Many hypotheses have been thought up over the years, and relatively few have been proved. Also, someone may come up with a good hypothesis that you may have to work out to prove. Be prepared to do some homework yourself.

GROUP ROLES

None

DOCUMENTATION FOR PORTFOLIO

None