

TITLE OF LESSON

Geometry Unit 1 Lesson 37 – Geometric Concepts: Polygons
Prove it! What's on the outside? What's on the inside? Of Geometry

TIME ESTIMATE FOR THIS LESSON

One class period

ALIGNMENT WITH STANDARDS

California – Geometry

12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

MATERIALS

None

LESSON OBJECTIVES

- To look at theorems and proofs related to polygons
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FOCUS AND MOTIVATE STUDENTS

- 1) Homework Check – Stamp/initial complete homework assignments. Pass back graded work and have students place in the appropriate sections of their binders.
- 2) **Agenda** – Have students copy the agenda.
- 3) Homework Review – (5 minutes) Review homework from Lesson 36. Recall the homework assignment was:

For each of the following make a drawing and choose one vertex. Draw all of the diagonals from this vertex. How many triangles are formed? What is the total sum of the measure of the interior angles for each figure? Challenge students, as a bonus, to come with a formula for the sum of the measure of the interior angles for a figure that has N sides.

- Quadrilateral (4-sided)
- Pentagon (5-sided)
- Hexagon (6-sided)
- Heptagon (7-sided)
- Octagon (8-sided)
- Nonagon (9-sided)
- Decagon (10-sided)

Have a different student draw each of the figures.

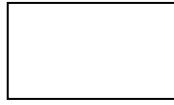
Two other students should answer the questions regarding the number of triangles and the total sum of angles.

Then collect the homework.

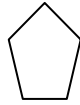
ACTIVITIES – INDIVIDUAL AND GROUP

1. Discussion – (3 minutes) Did anyone come up with a formula that determines the sum of the measure of the interior angles for a polygon with N sides?
Walk them through their answers for each of the figures below. Ask them to write down the number of sides ($N = ?$), the number of triangles formed ($\Delta = ?$), and the sum of the measure of the angles (number of triangles * 180°)

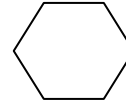
180, because each triangle measures 180° , so the number of triangles times 180 will give you the total measure of angles in the shape).



N = 4
2 triangles
 $2 * 180^\circ$



N = 5
4 triangles
 $4 * 180^\circ$



N = 6
4 triangles
 $4 * 180^\circ$

As you walk through, ask them to look for a pattern that they see throughout. The number of triangles is always what number less than the number of sides of a figure?

- Discussion – (2 minutes) Now let us look at a 5-sided figure. What is a 5-sided figure called? (pentagon) Have someone draw a 5-sided figure on the board. Now he should choose one vertex. Ask him how many diagonals he can draw from this vertex? (2) Have someone draw the diagonals. If N is the number of sides what is N for this figure? (5) Do you know how many diagonals can be drawn from this vertex, in terms of N? (Well, $N - 3$ and $N = 5$, so $5 - 3 = 2$.) When you draw all possible diagonals from a given vertex (i.e., 3), how many triangles do you divide the pentagon into? (3) Can you express this in terms of N? (Number of triangles = $N - 2$.) Does anyone want to try to express the sum of the measure of angles for this polygon in terms of N? (Sum of angles = $180 * (N - 2)$ —but only put this here if someone comes up with it. Otherwise, leave it. They'll get to it in a few minutes.)
- (10 minutes) Repeat this exercise with 6-sided, 7-sided, 8-sided, 9-sided, and 10-sided figures.
- Lecture – (5 minutes) Write the following on the board. Have students copy the theorem into the *theorems and postulates* section of their binders.

Theorem: *Sum of the Measure of the Interior Angles of a Polygon*

The sum of the measures of the interior angles of a polygon having N sides is $180^\circ(N - 2)$.

This means that, for example, a triangle has 3 sides, or $N = 3$. Therefore the sum of the interior angles of a triangle is $180^\circ(3 - 2) = 180^\circ * 1 = 180^\circ$. Is this true? Yes!

If we look at a square or any other quadrilateral we have one more side than a triangle so the formula for a quadrilateral, where $N = 4$, is $180^\circ(4 - 2) = 180^\circ * 2 = 360^\circ$. And we also know this is true.

- Lecture – (10 minutes) Write the following on the board and have the students copy the definition into the *terms and definitions* section of their binders.

An Exterior Angle of a polygon is an angle adjacent to an interior angle such that their exterior sides form a straight line.

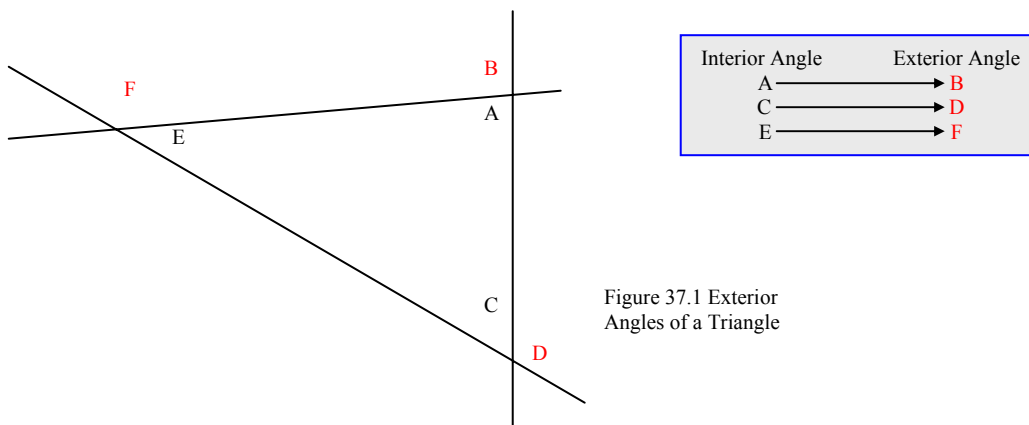


Figure 37.1 Exterior Angles of a Triangle

Draw figure 37.1 on the board. In this case the polygon is a triangle.

Since an exterior angle of a polygon is an angle adjacent to (or next to) an interior angle and their exterior sides form a straight line, we can look at a particular interior angle and then decide what the exterior angle is.

Have the students use string if it will help here – and it probably will. Be sure to have the students extend the string beyond the vertices for this activity, because we are also concerned with the area *outside* the triangle here. Ask someone to choose an interior angle. (Assume the answer is C although the possible choices are C or A or E)

Given that the choice is C, what is the exterior angle? Well, have them look at the definition. The exterior angle is adjacent to the interior angle and the exterior angle will form a straight angle when added to the interior angle. So in this case the exterior angle is D. Someone may want to use the other exterior angle near C (the unnamed one). That's fine, but label it. In fact, for each interior angle, there are two exterior angles. This is true for all polygons. Today, we'll focus on one to simplify things for students.

What about the other two interior angles? What are the associated exterior angles? (Interior angle A has B as the associated exterior angle and interior angle E has F as the associated exterior angle.)

Let us now look at a quadrilateral.

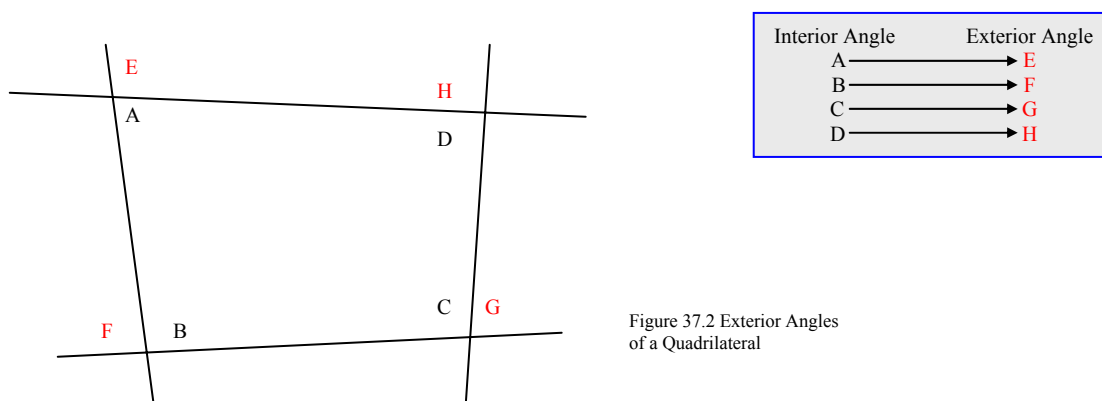


Figure 37.2 Exterior Angles of a Quadrilateral

Draw figure 37.2 on the board. In this case what are the interior angles? (A, B, C, D) What are the associated exterior angles? (E, F, G, H)

- Lecture – (5 minutes) Write the following theorem on the board and have the students copy it into the *postulates and theorems* section of their binders.

Theorem: Exterior Angle of a Triangle Theorem
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Can you show this to be true?

Proof: (Using Figure 37.1)

Statement	Reason
1. $m\angle C + m\angle A + m\angle E = 180^\circ$	
2. $m\angle C + m\angle D = 180^\circ$	
	3. $m\angle C + m\angle A + m\angle E = m\angle C + m\angle D$
	4. $m\angle A + m\angle E = m\angle D$
	1. Sum of the angles of a triangle = 180° .

2. Definition of an exterior angle tells us that an interior angle + and exterior angle form a straight angle and the measure of a straight angle = 180° .
3. Substitute statement 2 into statement 1.
4. Subtraction $m\angle C$ from both sides.
4. Lecture – (5 minutes) Write the following theorem on the board and have the students copy the theorem into their binders.

Theorem: *Sum of the Exterior angles of a Polygon*

The sum of the measures of the exterior angles of any polygon (one exterior angle for each vertex) is 360° .

Go through the following proof with the class:

Proof:

Statement	Reason
1. The sum of the interior angles for a polygon = $180 * (N - 2)$	1. The sum of the measures of the interior angles of a polygon having n sides is $180^\circ(N - 2)$.
2. The sum of the interior angles + the sum of the exterior angles = $180 * N$	2. Each interior angle plus its exterior angle sums to 180° and there are N of each.
3. $180 * N - (180 * (N - 2)) = 180 * 2$	3. Simplification of the formula for the sum of all angles (interior + exterior) – the sum of interior angles = sum of the exterior angles.
4. $180 * 2 = 360^\circ$	4. Multiplication
5. $360^\circ =$ the sum of the exterior angles	5. Transitive property

5. Homework – (5 minutes) Have students copy down homework.

HOMEWORK

- 1) Find the sums of the measures of the interior angles of a polygon having:
 - a. Four sides
 - b. Five sides
 - c. Six sides
 - d. Seven sides
 - e. 102 sides
- 2) Find the measure of the remaining angle for a polygon whose other angles measure:
 - a. Triangle: (12° , 90°)
 - b. Quadrilateral: (12° , 100° , 100°)
 - c. Pentagon: (112° , 142° , 91° , 92°)
 - d. Hexagon: (9° , 150° , 85° , 142° , 81°)

GROUP ROLES

None

DOCUMENTATION FOR PORTFOLIO

None