

TITLE OF LESSON

Geometry Unit 1 Lesson 36 – Geometric Concepts: Polygons, Quadrilaterals
Prove it! What's on the outside? What's on the inside? Of Geometry

TIME ESTIMATE FOR THIS LESSON

One class period

ALIGNMENT WITH STANDARDS

California – Geometry

13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

MATERIALS

None

LESSON OBJECTIVES

- To look at properties and theorems related to polygons
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FOCUS AND MOTIVATE STUDENTS

- 1) Homework Check – Stamp/initial complete homework assignments. Pass back graded work and have students place in the appropriate sections of their binders.
- 2) **Agenda** – Have students copy the agenda.
- 3) **Review** – (5 minutes) Review homework from Lesson 33. Recall that the homework from Lesson 33 was:

Look for examples in the world of the following. Come up with ten examples total. The more diverse your examples the better. Finding one of each will be the best. You can draw the example, cut the example out of a newspaper or magazine, or write a description of the example.

- | | | |
|--------------------|-----------------|-----------|
| • Polygons | • Quadrilateral | • Octagon |
| • Convex polygons | • Pentagon | • Nonagon |
| • Concave polygons | • Hexagon | • Decagon |
| • Triangle | • Heptagon | |

Starting with the convex polygon and continuing through the list, ask 3 or 4 students to give an example of each. They should show their drawings, read their descriptions, or show the clippings. Collect all homework assignments.

ACTIVITIES – INDIVIDUAL AND GROUP

1. Diagonals – (10 minutes): Draw the following on the board. Ask what this is. (Polygon, pentagon, convex polygon)

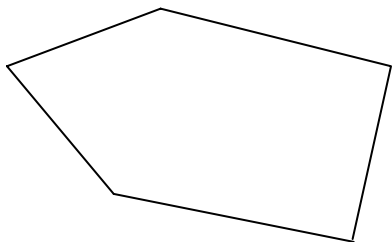


Figure 36.1 Pentagon

Write the following definition on the board:

A diagonal is defined as a segment joining any pair of non-adjacent vertices (any vertices that are not right next to each other).

Now, have five students stand in locations that form this polygon and designate one of the students as vertex A. Hand the student posing as vertex A two long pieces of string and ask, based on the definition on the board, if she can pass the other ends of these to two different vertices to create two diagonals. Ask, if they hold the string between two adjacent vertices (two right next to each other), what have they formed? (A side.) Wait until they create a form that looks like the drawing below, then have a student draw this on the board.

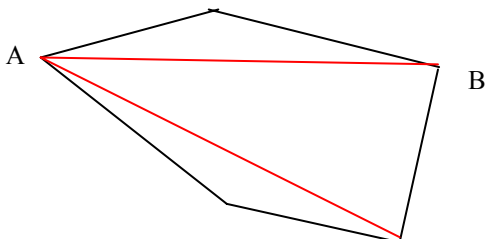


Figure 36.2 Two Diagonals of a Pentagon
Drawn from a Single Vertex

Ask the class if they can find any other diagonals that can be formed from vertex A. They can look at the drawing on the board or try to manipulate the student formation. They should discover that they cannot. Any other line they try to draw between two vertices will only give them a side.

Now, ask a student not in the formation to pick another vertex (student) and label her B. Now, ask for a volunteer to try to draw the diagonals from this vertex before the group can form them with the string. How many vertices were they able to create? (Two.) Look at a third vertex and don the same. Again, how many vertices can be drawn from that one? (Two.) Anyone want to take a guess, if we were to try a fourth vertex, how many vertices we'd be able to make?

- Equation – (5 minutes) If each vertex has two diagonals drawn from it and there are 5 vertices what is the total number of diagonals? (On the board write: $2 * 5 = 5$) Wait to see if anyone reacts. *What do you mean it's not 10? We all know that $2 * 5 = 10$ not 5!* But we have to notice that the diagonal between A and B is the same diagonal as the one between B and A. So we will be counting twice if we just multiple $2 * 5$, so the real answer is $(2 * 5)/2$ Write this on the board and erase the incorrect equation. Tell them the equation is (The number of diagonals drawn from a single vertex * the number of vertices)/2. Tell them to write this down and call it: *The equation to determine the number of diagonals in a polygon.*
- Demonstration – (7 minutes) Let's try it with a quadrilateral. Have four different students volunteer to form a square or a rectangle or some form of quadrilateral.

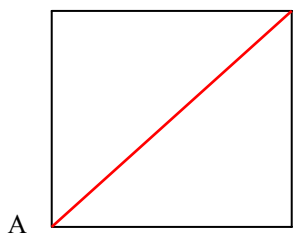


Figure 36.3 One Diagonal of a Quadrilateral
Drawn from a Single Vertex

Repeat the process as with the pentagon. Note that the (number of diagonals drawn from a single vertex * the number of vertices)/2 = 2 so that in the case of a quadrilateral we have a total of two diagonals.

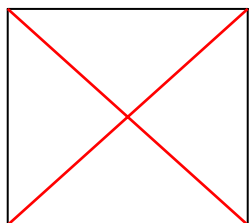


Figure 36.4 All of the Diagonals (2)
of a Quadrilateral

Repeat the same process with a hexagon. (The number of diagonals drawn from a vertex * the number of vertices)/2 = $(3 * 6) / 2 = 9$

4. Discussion – (10 minutes) Ask if anyone notices a pattern here. How many diagonals can you draw from each vertex in a quadrilateral? (1.) How many diagonals can I draw from each vertex in a pentagon? (2.) How many diagonals can I draw from each vertex in a heptagon? (3.) What about at triangle? (0.) What do you think the answer is for a decagon? (7.)

This is because the number of diagonals that can be drawn from a single vertex in any given polygon = $N - 3$, where N is the number of sides (or vertices since they are equal to each other). How many diagonals can be drawn from a single vertex in a shape with 1,000 sides? (997.) This is how you determine the number of diagonals that can be drawn from **each** vertex.

On the other hand, to determine the total number of diagonals that can be drawn for a shape, the formula is (the number of diagonals from a single side * the number of sides)/2. So, if a polygon has N sides then $\frac{1}{2} * N(N - 3)$ is the total number of vertices that can be drawn for a polygon. What is the total number that can be drawn for a decagon? ($\frac{1}{2} * 10 * 7 = 35$). What is the total number that can be drawn for a 12-sided figure (dodecagon)? ($\frac{1}{2} * 12 * 9 = 54$)

5. Concept Review – (10 minutes) So, let's make some connections with what we now know:

What is the sum of angles in a triangle? (180°)

What is the sum of angles in a square or any quadrilateral? (360°)

How do you know that? (We know that a square has four angles each of which is 90° .)

We know that a square has 360° but do we know that all quadrilaterals have 360° ? (Yes.)

Draw a quadrilateral on the board. Ask the students to choose one vertex. Ask someone to draw all the diagonals from that vertex. (There is only one)

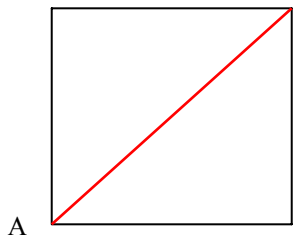


Figure 36.5 One Diagonal of a Quadrilateral Drawn from a Single Vertex

Drawing this vertex divides the quadrilateral into 2 triangles. If a quadrilateral can be divided into two triangles and each triangle has 180° then what is the total measure of the angles in the quadrilateral? ($2 * 180^\circ = 360^\circ$.) This is great practice for connecting things that they hadn't previously thought to connect. This will be very useful as they get into more complicated theorems.

6. Homework – (2 minutes) Have students copy their homework.

HOMEWORK

For each of the following make a drawing and choose one vertex. Draw all of the diagonals from this vertex. How many triangles are formed? Challenge students, as a bonus, to come with a formula for the sum of the measure of the interior angles for a figure that has N sides.

- Quadrilateral (4-sided)
- Pentagon (5-sided)
- Hexagon (6-sided)
- Heptagon (7-sided)
- Octagon (8-sided)
- Nonagon (9-sided)
- Decagon (10-sided)

Prove It
How do we create truth?

2:1:36:Geometric Concepts: Polygons, Quadrilaterals

GROUP ROLES

None

DOCUMENTATION FOR PORTFOLIO

None