THE LANGUAGE OF MATHEMATICS
Making the Invisible Visible

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Symbols of progress

The first systematic use of a recognizably algebraic notation in mathematics seems to have been made by Diophantus, who lived in Alexandria sometime around A.D. 250. His treatise *Arithmetic*, of which only six of the original thirteen volumes have been preserved, is generally regarded as the first 'algebra textbook'. In particular, Diophantus used special symbols to denote the unknown in an equation and to denote powers of the unknown, and he employed symbols for subtraction and for equality.

These days, mathematics books tend to be awash with symbols, but mathematical notation no more *is* mathematics than musical notation *is* music. A page of sheet music *represents* a piece of music; the music itself is what you get when the notes on the page are sung or performed on a musical instrument. It is in its performance that the music comes alive and becomes part of our experience; the music exists not on the printed page, but in our minds. The same is true for mathematics; the symbols on a page are just a representation of the mathematics. When read by a competent performer (in this case, someone trained in mathematics), the symbols on the printed page come alive—the mathematics lives and breathes in the mind of the reader like some abstract symphony.

Given the strong similarity between mathematics and music, both of which have their own highly abstract notations and are governed by their own structural rules, it is hardly surprising that many (perhaps most) mathematicians also have some musical talent.

In fact, for most of the two and a half thousand years of Western civilization, starting with the ancient Greeks, mathematics and music were regarded as two sides of the same coin: both were thought to provide insights into the order of the universe. It was only with the rise of the scientific method in the seventeenth century that the two started to go their separate ways.

For all their historical connection, however, there was, until recently, one very obvious difference between mathematics and music. Though only someone well trained in music can read a musical score and hear the music in her head, if that same piece of music is performed by a competent musician, anyone with the sense of hearing can appreciate the result. It requires no musical training to experience and enjoy music when it is performed.

For most of its history, however, the only way to appreciate mathematics was to learn how to 'sight-read' the symbols. Though the structures and patterns of mathematics reflect the structure of, and resonate in, the human mind every bit as much as do the structures and patterns of music, human beings have developed no mathematical equivalent of a pair of ears. Mathematics can be 'seen' only with the 'eyes of the mind'. It is as if we had no sense of hearing, so that only someone able to sight-read musical notation would be able to appreciate the patterns and harmonies of music.

In recent years, however, the development of computer and video technologies has to some extent made mathematics accessible to the untrained. In the hands of a skilled user, the computer can be used to 'perform' mathematics, and the result can be displayed in a visual form on the screen for all to see. Though only a relatively small part of mathematics lends itself to such visual 'performance', it is now possible to convey to the layperson at least something of the beauty and the harmony that the mathematician 'sees' and experiences when she does mathematics.